Experimental Tests of the Spin-Statistics Connection and the Symmetrization Postulate

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Outline

- Background
- Q-mutators and possible violations of the spin-statistics connection
- Experimental Tests
- Composite Systems
- Conclusions and Outlook

Symmetrization Postulate

- Quantum states of identical particles are either symmetric or anti-symmetric under the interchange of particle labels.

- Trivial for two-particle states but a significant limitation for N > 2.

Spin-Statistics Connection

- Fermions (anti-symmetric states) have spin quantum numbers of 1/2, 3/2, ...
- Bosons (symmetric states) have spin quantum numbers of 0, 1, 2, ...
- Spin-Statistics “Theorem”
  - Pauli 1940
  - Many others 1950s
  - Relativistic Quantum Field Theory +

->SSC is compatible with QFT

Statistics of Composites

- A composite of an odd number of fermions behaves like a fermion
- Otherwise, the composite behaves like a boson.
- Examples:
  - H-atom, ^{23}Na, $^{85,87}Rb$ - “bosons”
  - ^{40}K atom - fermion
  - ^{16}O nucleus - boson

Fundamental Principle

- If the particles are identical, observable results should not depend on how we label the particles.
- Permutation symmetry: observables are unchanged under permutation of the identical particle labels.
- (not physical exchange of particles)

Fundamental Theorem

- For states with different permutation symmetries

$$\langle \Psi_1 | \hat{V} | \Psi_2 \rangle = 0$$

Proof:

$\hat{V}$ unchanged by permutation

of identical particle labels

Þ $\langle \Psi_a | \hat{V} | \Psi_s \rangle = - \langle \Psi_a | \hat{V} | \Psi_s \rangle$

Consequences

- Permutation Symmetry of a system does not change with time
- Transitions between states of different permutation symmetry are strictly forbidden.
- Superselection Rule

Types of Experimental Tests of the SSC (spin-statistics connection)

- Transitions between “SSC-forbidden” energy levels
- Accumulation of particles in SSC-forbidden states, e.g. atomic Li with all three electrons in the 1s orbital
- Deviations from standard fermion/boson statistics in bulk systems

Tests of the Symmetrization Postulate

- Need to look at systems with N > 2 identical particles
- Search for states associated with higher dimensional representations of the permutation group
- Possibilities: NH_3, OsO_4 etc.

How to Characterize Experimental Tests

- Density matrix formulation

Two-particle state: s = symmmetric, a = anti-symmetric

$$\rho^{(2)} = A_s^{(2)}\rho_s^{(2)} + A_a^{(2)}\rho_a^{(2)}$$

Three-particle state

$$\rho^{(3)} = A_s^{(3)}\rho_s^{(3)} + A_a^{(3)}\rho_a^{(3)} + A_{m1}^{(3)}\rho_{m1}^{(3)} + A_{m2}^{(2)}\rho_{m2}^{(3)}$$

Two 2-dimensional reps.

Q-mutators

- O. W. Greenberg, 1990

$$a_k a_j^\dagger - q\, a_j^\dagger a_k = \left[a_k , a_j^\dagger \right]_q = \delta_{kj}$$

q-mutator

$q = +1$ bosons

$q = -1$ fermions

$-1 < q < +1$ "quons"

q-mutators: Interpretation

- In the q-mutator formalism

$$A_s^{(2)} = \frac{1+q}{2} \qquad A_a^{(2)} = \frac{1-q}{2}$$

$$A_s^{(3)} = \frac{(1+q)(1+q+q^2)}{6} \qquad A_a^{(3)} = \frac{(1-q)(1-q+q^2)}{6}$$

$$A_{m1}^{(3)} = \frac{(1+q)^2(1-q)}{3} \qquad A_{m2}^{(3)} = \frac{(1+q)(1-q)^2}{3}$$

Transition amplitudes are proportional to (1+q)/2 etc. after you take normalization into account.

Experimental Tests

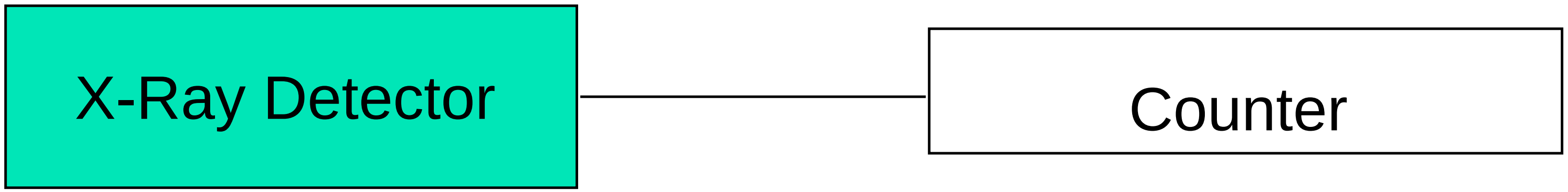
- Electrons
- Nuclei
- Photons
- Symmetrization Postulate

Tests for electrons

- Bulk matter electrons
- Atomic electrons

A Non-Test of the Spin-Statistics Connection

Reines and Sobel, PRL 32, 954 (1974)

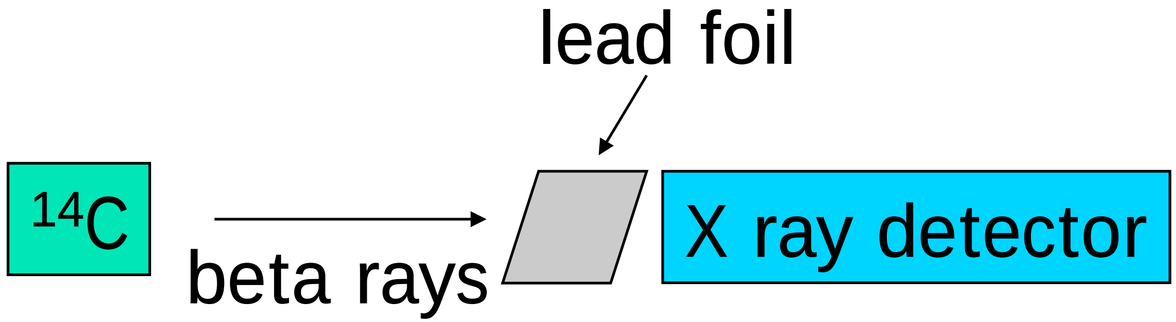


$(-1+q)^2 < 10^{-22}$

But, this analysis violates the Fundamental Theorem:

$$\langle \Psi_s | \hat{V} | \Psi_a \rangle = 0$$

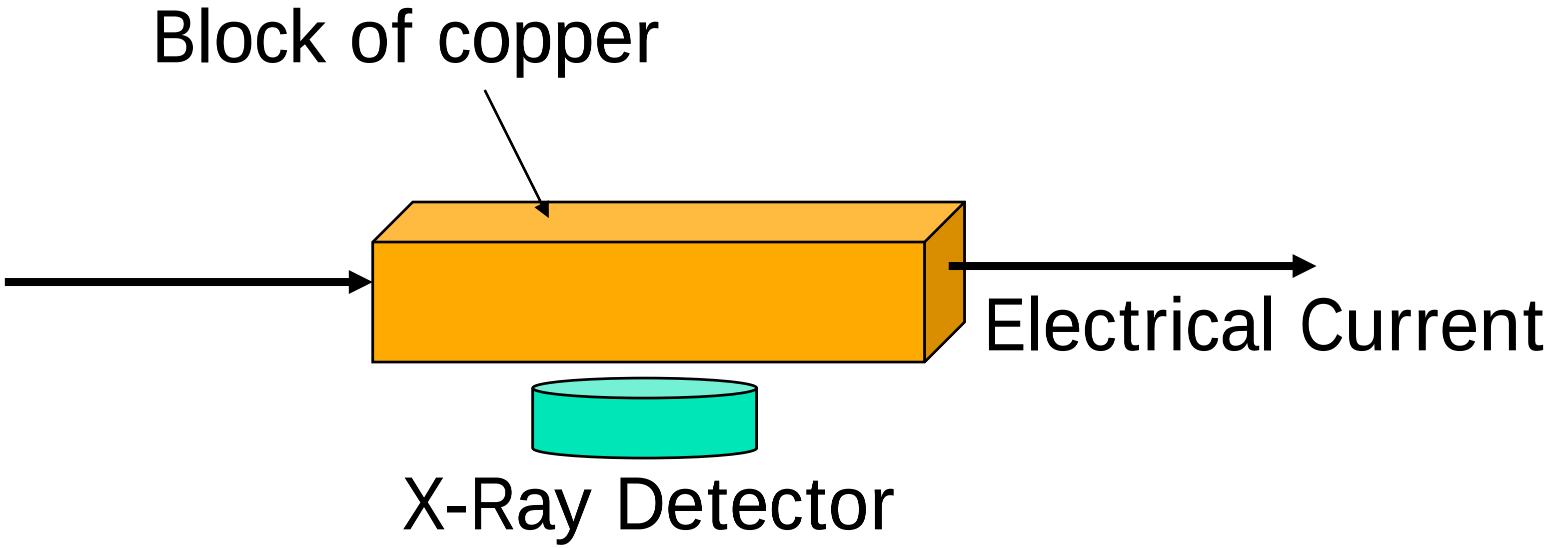
Goldhaber and Scharff-Goldhaber 1948



Original question: Are beta rays “identical” to electrons?

Reinterpret as test of the Pauli Exclusion Principle

Ramberg-Snow Experiment



Results: probability of making a transition to already occupied state $< 10^{-26}$

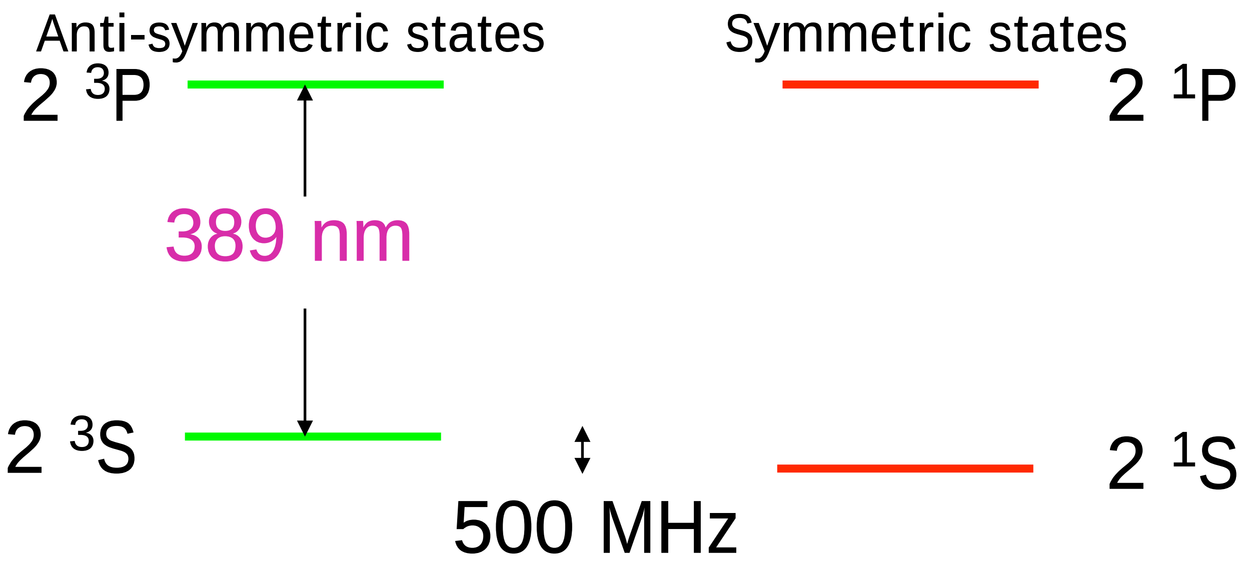
Updated version: VIP Experiment, Pietreanu and colleagues

Atoms in SSC-violating States

- Example Be with $(1s)^4$ in place of $(1s)^2$ $(2s)^2$
- High Precision Mass Spectrometry
- D. Javorsek, et al Phys. Rev. Lett. (2000) $[Be'] < 10^{-11} [Be]$

Energy Levels in Atomic Helium Test for Electrons

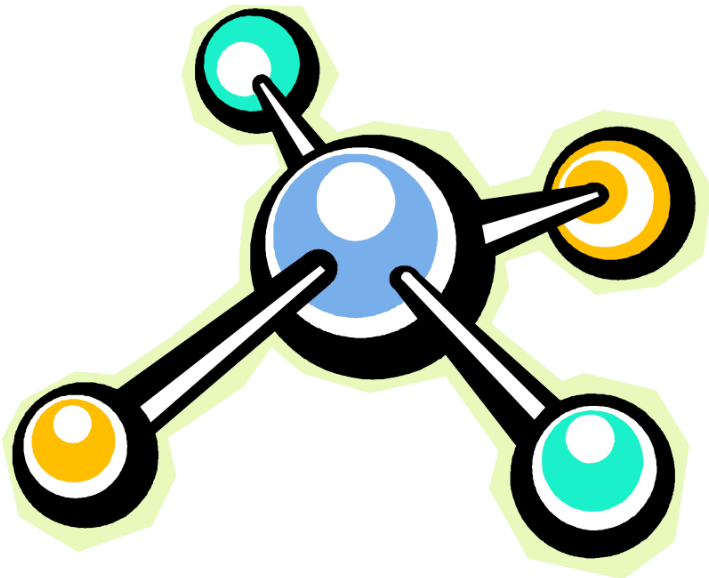
- Deilamian, Gillaspy, and Kelleher 1995
- Atoms excited in electrical discharge



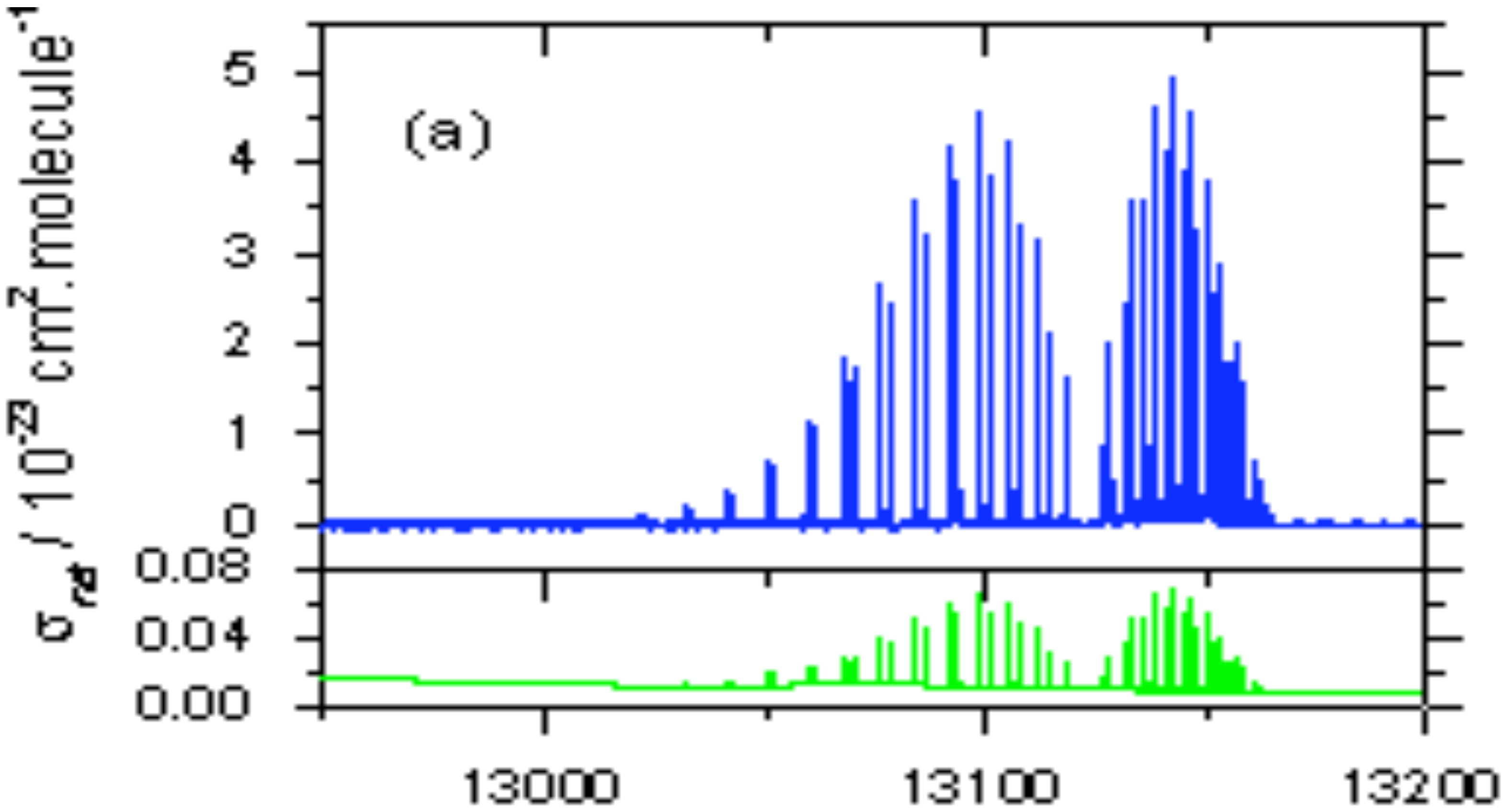
•SSC Forbidden < 10^{-6} SSC Allowed

Molecular Spectroscopy Tests for Nuclei

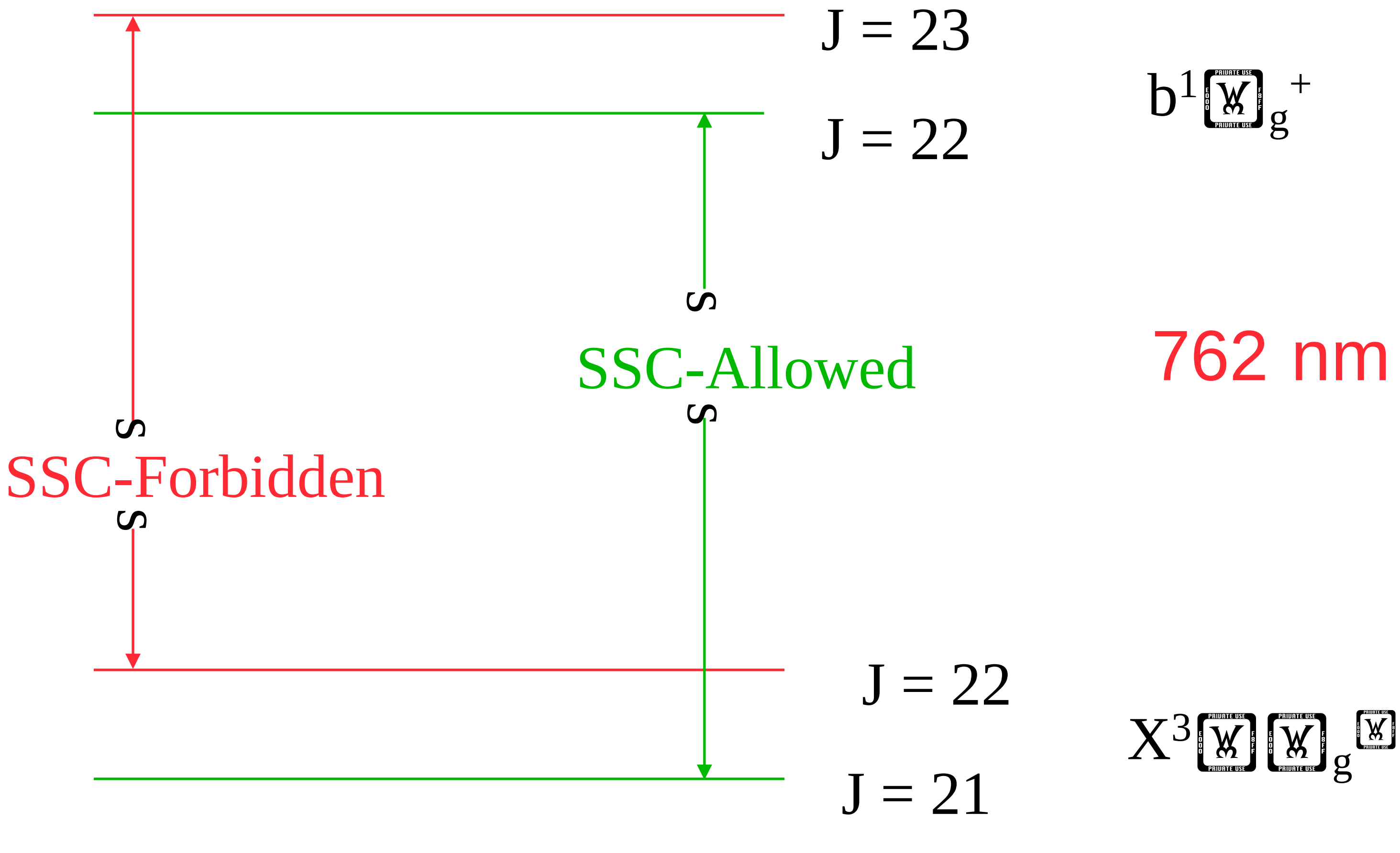
- Back to the beginning!



O_2 Spectrum near 762 nm



Molecular Oxygen
^{16}O-^{16}O (nuclear spin = 0)



^{16}O results

- O_2
  - Hilborn and Yuca (PRL, 1996)
  - Tino et al (PRL, 1996)
  - $(1-q)^2 < 5 \times 10^{-6}$
- CO_2
  - Modugno et al (PRL, 1998, 2000)
  - $(1-q)^2 < 1 \times 10^{-11}$
  - Lien talk

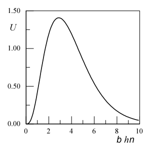
Tests for Photons

- Planck Distribution for Thermal Radiation
- J = 0 to J = 1 two-photon transition
- Rydberg Atoms and Cavity QED

Thermal Radiation

$N = 2?$ $\quad$ [illegible] $= 1/k_BT$

$N =$ [illegible]$?$



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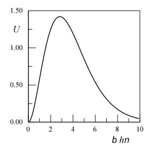


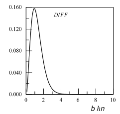
B

Thermal Radiation

Partition Function:

$$Z_N = \sum_{n=0}^{N} e^{-\beta h\nu n} = \frac{1 - e^{-(N+1)\beta h\nu}}{1 - e^{\beta h\nu}}$$

Difference for $N = 2$



Mean occupation number:

$$\overline{n} = \frac{\sum_{n=0}^{N} n e^{-\beta h\nu n}}{Z_N}$$

density of modes: $\frac{8\pi\nu^2}{c^3}$

Two-Photon Transition Between J = 0 and J = 1 States in Atoms

- Budker, Demille, Brown, English et al

$J = 1$

Landau-Yang Theorem (1948 and 1950)

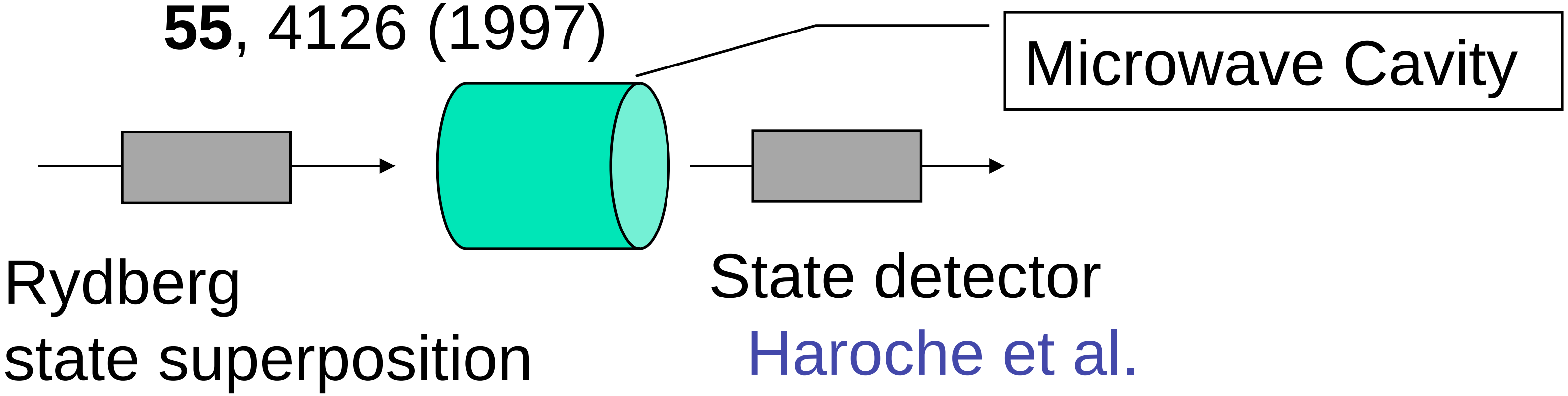
$J = 1 \not\to 2$ photons if photons are pure bosons

$J = 0$

Particle physics experiments not very limiting.

Another Photon Test

C. G. Gerry and R. C. Hilborn, Phys. Rev. A **55**, 4126 (1997)



1. Detecting first atom in a certain state leaves the cavity photons in an "even" or "odd" coherent state.
2. Probability of finding the next atom in the same state = 1 if the photons are pure bosons.

P_{diff} [illegible] $(1 - q)^2$

Experimental Tests of the Symmetrization Postulate

- Need systems with $N > 2$ identical particles.

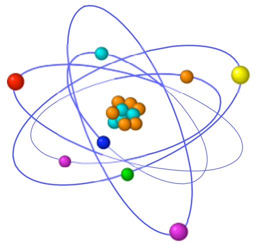
Polyatomic Molecules

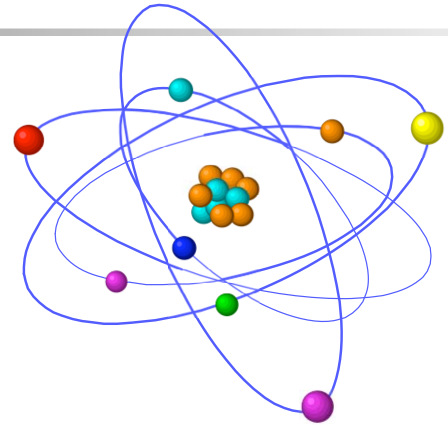
- With three or more particles of the same type, the possibility of higher-order permutation symmetries (beyond symmetric and anti-symmetric)
- Higher-dimension representations of the permutation group

Polyatomic Molecules

- Christian Borde OsO_4 (discussed by G. Tino, Modugno, Inguscio, et al)
- **The spin-vibration hyperfine interaction in the nu3 band of 189OsO4 and 187OsO4: a calculable example in high-resolution molecular spectroscopy**,
- **C.R. Physique 5, 171-187 (2004).**

Composite Systems





- Wigner (1929); Ehrenfest and Oppenheimer (1931):
  - a composite with N fermions is a fermion if N is odd, otherwise a boson.
- Greenberg and Hilborn (PRL,1999)- what if spin-statistics violated?

Quon Composites

For a composite of *N* identical particles

$$q_{composite} = (q_{constituent})^{N^2}$$

^{16}O nuclei in CO_2: 1- q < 3 x 10^{-6} Modugno, Ingusicio, and Tino, Phys. Rev. Lett. 1998, 2000

i.e. probability proportional to $(1-q)^2$

- for nucleons 1-|q| < 1 x 10^{-8}
- for quarks 1-|q| < 1 x 10^{-9}

Conclusions

- SSC is consistent with QFT, but what about M-theory, supersymmetry, quantum gravity, etc. ??? Need some theory!
- Most experiments testing SSC are still rather crude.
- Experimental limits on violations of the SSC for composites can be used to set even lower limits for the constituents.